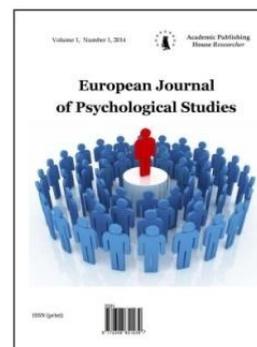


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### **Seven Paradoxes of Voting Systems: Pitfalls in Collective Decision Making**

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#### **Abstract**

Some peculiarities of a group decision making are discussed. Certain undesirable but unavoidable features are inherent to all voting procedures, i. e. to the elections.

**Keywords:** voting; elections; collective decision making.

#### **Introduction**

Choice, or decision making, is a critical operation in any activity of a human entity – will it be either one person, or several persons in a group, or pack of individuals in organization larger than what we normally think of as a group. The vital importance of choice has had attracted attention of many researchers and thinkers, and by now one can say that knowledge and understanding of choice nature form a “theory of decision making“.

The most general definition of choice is as *the purposeful tapering (narrowing) of a set (variety) of alternatives*. The great variety and diversity of components of a choice procedure, - sets of alternatives, human entities (subjects) making decision, and purposes, needs and desires of the individuals, has led to a tree-like construction of the theory. One of its branches is the theory of multi-sided decision making, focusing on the specific effects emerging in cases when the final decision is a result of interweave of individual decisions of different persons. This branch has its twigs corresponding to various degrees of concordance between the decision makers. The subject of this paper is the case when a common consent exists between individuals about their ultimate goal, but their opinions about preferences between alternatives (means of achieving the goal) differ.

#### **Procedures of collective decision making**

Let us focus our attention on the election procedures only.

Let  $X = \{x_1, x_2, \dots, x_k\}$  be a ballot-list with names of candidates. Let the electorate consist of  $N$  voters. Let  $C_i(X)$  be a “choice” of  $i$ -th voter on the set of alternatives. The function  $C_i(X)$  describes preference of  $i$ -th voter, and it may be either just  $x_i$ , a name of the most preferable candidate, or an enumeration of all candidates in the order of preference (if it is requested by rules of voting).

After act of voting we have a set of  $N$  voting-papers with voters’ choices in them :  $C_1(X), C_2(X), \dots, C_N(X)$ . If all voters have the same preference, there is no problem. But as a rule, they are different. Then we have problem: How to define the “common choice” when we have a collection of different personal choices?

This may be done by using a certain procedure of converting a set  $\{C_i(x)\}_{I^n}$  into one “common” choice  $C_o(X)$ :

$$C_o(X) = f\{C_1(X), C_2(X), \dots, C_i(X), \dots, C_N(X)\}. \quad (1)$$

Here  $f$  is an algorithm of the conversion. This function is called *an election procedure*.

A question arises: what are properties of decisions obtained by an election procedure? This is a task of purely formal study of the formula (1), and this problem was attacked by several mathematicians of the last century [1].

The main difficulty stemmed from the fact that various election procedures can be applied to the set of voting–papers. After considering various particular cases some general and specific properties were found of the collective decision making. These findings represent something that can be named *a theory of voting systems*.

The fundamental discovery was that election procedures, besides of their commonly desirable property – to produce ultimate “common” decisions, possess and other intrinsic, latent, undesirable properties which sometimes make result of voting look as “strange”, “unjust”, “useless”, “incorrect”, etc. The main results of studying such cases may be presented as *seven paradoxes of voting systems* [2]. We will present them in the order typical of Russian folktales and American thrillers: the further a story goes, the more terrifying it is.

### Seven paradoxes of voting systems

**Paradox 1.** *The truth can not be found by voting.* In other words, a collective decision (even unanimously accepted) is not surely right and correct. The history knows many cases when one man possessed the truth and all the others were mistaken: Galileo with his rotating Earth, Jordano Bruno burned on a bonfire for his heresy, Kopernik, et al. The point is that each voter is a human being, a subject, whose characteristic (and distinctive from an object) property of ability to estimate anything is accompanied by a liability to mistakes and errors in his estimations. And a collective of individuals is also a human entity, a subject, -- with the same property. Was the collective right or wrong, this will be proved *after* the decision is implemented in reality.

There is one important virtue in voting: the collective decision is *statistically (!)* more reliable than the individual one. This is due to mutual compensation of extreme opposite opinions. It is especially clearly seen on the averaging quantitative measurements in statistics, which leads to decreasing the variance of the position parameter estimate.

Thus, the most important peculiarity of the voting is that although it is designed to approximate the truth, its main role in social systems consists in the coordinating actions of a group *after* making decision: every member of the group is obliged to follow the decision, even if he/she personally has voted against it.

**Paradox 2.** *Although the election procedure is intended to arrive to a decision, it may result in not making it.* The point is that any procedure presupposes a fulfillment of certain conditions. If any of them is not observed, the procedure loses its applicability, and the situation remains indefinite as before voting. And this is typical of *any* procedure of collective choice.

For instance, the procedure of “simple majority” prescribes accepting alternative by obtaining 50% + 1 vote. But if the even number of voters divides exactly in to equal parts – the decision is not made. Sometimes they decide to grant the chairman with more than one vote – to overcome this situation. But if the number of voters is odd, and voters have divided exactly to the “50%+1” proportion, then how to proceed if the chairman belongs to a lesser half? Was he given more or less than 1 additional vote?

In case of “qualified majority” of 2/3 (as in the rules of awarding with scientific or academic degrees by expert boards in Russia), the number of members attending the board meeting is often not a multiple of three, and sometimes a shortage of voices is a very small part of lacking one. Even the instruction from the Higher Attestation Commission, to round result in favor of the defender, did not fully removed ambiguity – to round or not to, if the shortage is more than one half?

Even a rule of “unanimous opinion” can hover, and that often happens in some organizations (parliaments, UNO, etc.) keeping to this rule.

**Paradox 3** (Condorcet’s). This paradox exhibits itself when preferences of votes make up a circle. For example, let three fractions in parliament come up with their own variants of a bill:  $a, b,$

and c. Or three chaps are arguing whose girlfriend is better. If any of these trios have their preferences as  $C_1=(a>b>c)$ ,  $C_2=(b>c>a)$ ,  $C_3=(c>a>b)$ , then they are in the paradox of Condorcet. Any procedure of voting will either finish without coming to a decision (because under such preferences there is no most preferable variant), or, – after applying a forcibly stopping rule (like the Olympic comparisons in pairs), – the result depends on the order of considering alternatives (see Fig. 1).

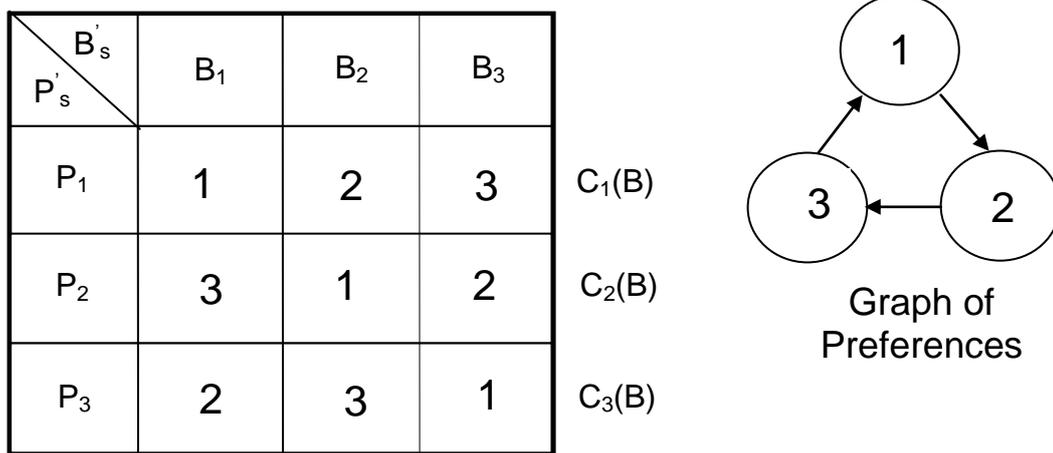


Fig. 1. The Condorcet Paradox

Sometimes the Condorcet paradox is not essential (for instance, when the cycle appears in the low part of the alternatives chain, it will not affect the choice of the leader). But if it is essential (i.e. all of the potential leaders are involved into the cycle), the way out of it is to induce one of voters to change his preferences between unimportant for him alternatives (this is called “black” or “white” PR). The cycle disappears, and the single solution appears (see Fig. 2.).

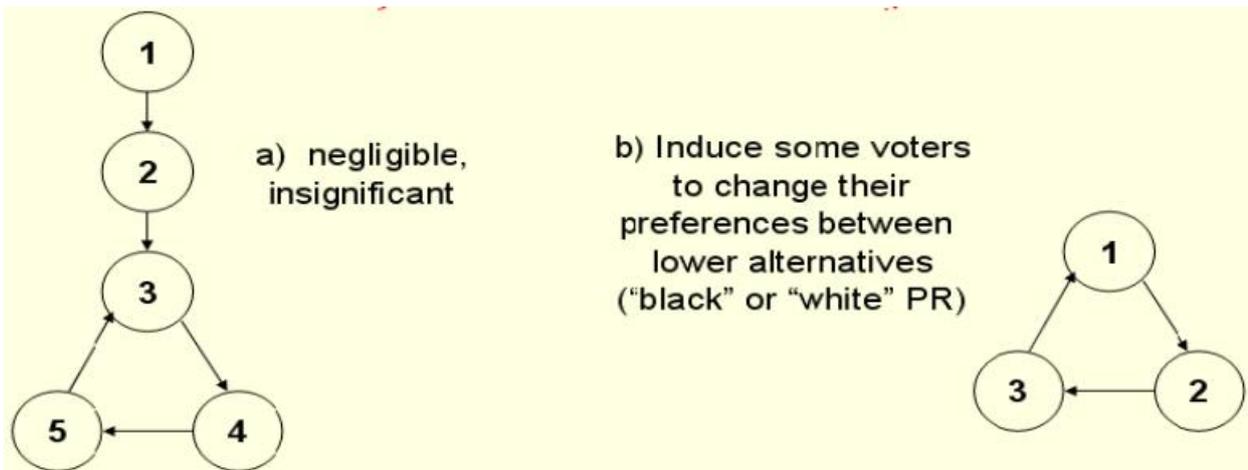


Fig. 2. Variants of the paradox

**Paradox 4.** *Under any majority voting procedure, the minority can win.* It turns out that there are many legitimate possibilities for a minority to win under the majority voting system. Let us consider some of them.

- First one is the recognition of elections to be legitimate even if the attendance of voters is lower than 50%. This automatically gives a right to make decisions to a minority of the electorate. Perhaps there is nothing wrong with it: your ignoring voting means that you are indifferent to outcome of elections, then let the interested minority to decide.

*But minority can win even under 100% attendance of voters, and by several different ways.*

- One of possibilities emerges with the “splitting votes”. For instance, let one party have 60% of electorate, and another one – 40%. If the first one nominates two equally good candidates, and the second one puts forward only one candidate, the last one wins and the major party loses. Reasons why the party of majority does this, could be various, but the result is fatal. (See Fig. 3).

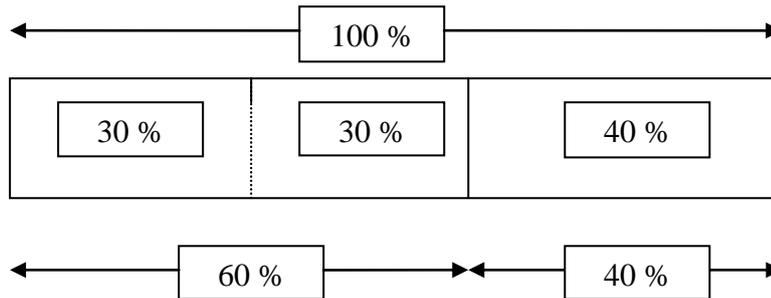


Fig. 3. Splitting votes

*But minority can win under 100% attendance even without splitting votes, and again – and, again, with options.*

- Let we have a majority voting system with a rule of 2/3. If in the end the minority has won, hence at the last stage of voting it obtained 2/3 of votes. But if every elector in the last stage was elected on the same 2/3 basis, then a situation is possible as shown at Fig. 4, when minority wins (in this example 4/9 against 5/9).

But for realization of such a possibility it is necessary to satisfy three conditions:

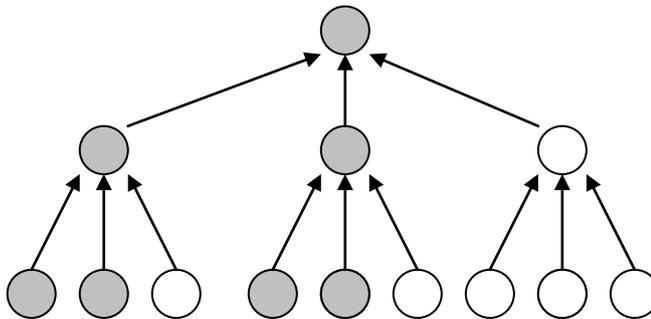


Fig. 4. Multy-staged elections

(1) elections must be multi-staged, and at each stage decision is made by majority, (2) minority must obey a discipline of voting imposed by their leaders (imagine what happens if one of voters would exchange places with the other party); (3) minority must be numerous enough – for ensuring its majority at the last stage (imagine that in our example a minority is not 4 but only 3 voters). But it must be noted that the necessary proportion of minority could be arbitrarily small, if the additional levels of elections are admissible. For instance if in our example to add one more level, then winning proportion of minority is descended from 4/9 (44.4%) to 8/27 (33.7%).

This scheme has not only a theoretical interest: multistage elections are used in real life. Say, two-staged election procedure of presidential elections in the USA already 4 times of 43 has brought to power candidates from minority (Bush beat Gore with only 48% of votes in the Primaries). The same scheme guaranteed to Soviet leaders being permanently elected for many years in a row, in spite the ballot was “direct and secret”.

*But minority can win even under one-stage elections!*

• An example of this gives an election procedure in Australia. Elections there are one-stages physically: every voter fills in a ballot-paper only once. But in contrast to other systems an elector must not simply nominate the most preferable candidate, but rank all candidates in the order of his/her preferences. The election procedure is as following. From all ballot-papers they extract candidates with the first priority. If any one of them collected majority – he becomes a member of parliament. But if there is no candidate with majority (remember splitting votes?), then they took candidates with second priority (do you see multistage procedure – not physically but algorithmically?) and again define whether somebody took the majority. And so on. What a party of minority is doing? They ask their followers to take part in voting and oblige them to put their candidate at the last line of the list. (Do you see a condition of voting discipline for a minority?) The expectation is that splitting votes between other parties at upper levels will bring the procedure to the bottom line, and we all are there, and majority of other voters have put us in the last priority. This gives us a majority of votes. This is why minor parties are presented in the Australian Parliament.

**Paradox 5 – of overwhelming majority.** Many believe that, under the election rule “one person – one vote”, the more votes were given to an alternative the more *democratic* is the decision made. This is a delusion! Perhaps this impression is based on the fact that politicians feel themselves the more sound and confident the larger part of electorate supports them, and to the larger extent they counted to be representative of people.

The paradox consists in psychologically understandable impression based on the prevalent concept of “Us and Them”, of “own and alien”, but *this has nothing to do with the concept of democracy.*

Let us show it on a simple example. Let the maximally “democratic” election procedure be accepted, with only two rules:

(1) Under any number  $N$  of voters, a decision is taken only if “pro”s are not less than  $N - 1$ , and “contra” is only one. (Note that  $N$  may be any, and even very large, number).

(2) Every voter is voting “pro” if the alternative is not harmful to him personally (and, of course, he does it with even more enthusiasm if it is beneficial to him).

It looks like impossible to suggest more “democratic” procedure. But if a society approves this procedure for making collective decisions, - it bids farewell to the democracy. From now on, the chairman may (if he wants to) realize by this procedure any decision he likes. The next example reveals it explicitly.

Let us apply the procedure to deciding whether we all shall pass from a state to another one. Let “a state” be the presence of a certain sum of money in everyone’s pocket. To pass to another state means to take some money from one person and to distribute it somehow between the others.

Then the theorem holds: It is possible to come, by this procedure, from any initial state to any pre-designed state by a finite number of steps.

To be definite, imagine that I would like to transfer all your money in one pocket. First step: “Who is for taking money from a (named) person and sharing it between all the others?”. The result is evident. To accelerate the process, the chairman can suggest to transfer all money from a (named) person directly to the target one. The procedure will work perfectly. Sooner or later, the purpose will be achieved, and in fully legitimate way.

Do not think that this example is artificial. What is worse, in practice the procedure may be accompanied by liquidation of those discontented, as during Stalin’s purges in the past century.

The essence of this paradox lays in the fact that this procedure makes legitimate sacrificing interests of one person for the sake of interests of all others. But the others ignore that everyone of them will become the next victim.

Thus, *voting by majority and democracy – are simply different things.*

Democracy has several definitions, including those containing general elections as an indication of it. But the essence of democracy is not concentrated in the secret and direct general elections (exactly that was one of the clauses of Stalin’s “the most democratic” Constitution). Decisions may be made collectively or individually; but *democracy consists in defending basic*

*human rights (on life, on property, on freedom) of any minority – in the process of implementing the decision.*

**Paradox 6 – of the unanimity.** If the key indication of democracy is a defense of interests of everyone, then the only democratic voting system is the unanimous decision-making: everyone can defend own interests already at the stage of making decision, just by saying “no” to a disliked alternative.

In fact, such procedure is applied in many highly responsible situations: the rule of veto in some parliaments; in Security Council of UNO; the next Pope is elected by cardinals only unanimously; the verdict of jurors; decision making in joint-stock companies with unlimited responsibility, – are just a few examples. In professional facilitating of the applied systems analysis (solving problems) it is strongly recommended to use this procedure in each act of collective choice [3].

But paradoxical situations happen under this procedure, too.

(1) Sometimes the unanimity principle (all are “pro”) is substituted by the principle of consensus (nobody is “contra”); but they are quite different things: the abstainers are treated identically to the “pro”s, the absent ones are not taken into account at all. A dramatic example was the decision of the UN Security Council to start military operations in Korea under the aegis of UNO – in absence of the Soviet representative.

(2) Second paradoxical situation emerges when a desired alternative can no-wise collect 100% of votes. There are at least three ways of trying to overcome this difficulty.

First one is a search for a compromise settlement, In Fig.5 it is seen that unanimity is impossible because an alternative acceptable for all parties does not exist (circles outline acceptable alternatives for each one of them). The way out of this may be in persuading someone to concede by slightly extending a set of acceptable alternatives, i.e. to agree on compromising (dotted line).

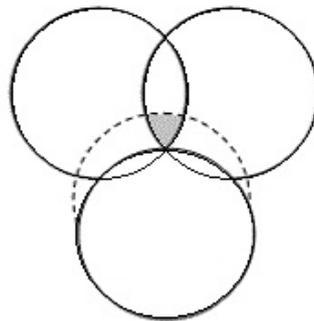


Fig. 5. Scheme of the one-sided compromise

The second way can be called “a ladder principle”. We can try to achieve the goal unattainable in one step, by doing one by one several small steps toward it, but each step – unanimously. (The idea is that small steps are easier agreed than the large one.) Sometimes it becomes evident on a middle step of the “ladder” that the pursued final solution does not worth to make (like in Ackoff’s example with the bill on the capital punishment for murder [4]),

The third way – to stop making unsuccessful collective voting and to apply to an authoritative and respectful person for his decision or advice.

**Paradox 7: The Arrow’s Theorem of Impossibility.** Arrow has obtained the most general theoretical results about collective choice, and was awarded by Nobel Prize in Economics for his findings. The most famous of them became his “Theorem of Impossibility”. The question was: “Could something consistent be said about all voting procedures?” (See formula (1) at a beginning of the paper). The answer was “No” to a ll procedures; but one can try to say something definite about procedures “good”, “acceptable”, “reasonable”, i.e. satisfying certain sensible conditions.

The reasonable conditions, expressing our understanding of what is the “right”, “just”, “good” choice, were only four (in the original paper there are some other ones, but they are of purely technical – about numbers of alternatives and voters):

(1) All individual choices  $C_i(X)$  have to be taken into account *somehow*, not compulsory with equal weights, but the extreme form,  $C_o(X) = C_i(X)$  (“the dictator’s choice”), is *undesirable*.

(2) *Condition of monotony of  $f(.)$* . Any alternative, accepted by a certain number of votes, must not be rejected by a larger number of votes in its favor.

(3) *Condition of independency of alternatives*. If changes in an individual preferences did not touch on certain alternatives, then in a new collective ordering they must keep their ranks.

Let us give an illustrating example to this condition. Let us come back and extract a one voting-paper from the ballot-box and return it to the voter “for second thinking”. He changes his mind about two alternatives and swaps their places in his list. The corrected paper is returned in the urn and the counting is repeated, It might (usually not) change the final result, but if it does (in case of unstable situation when one vote makes the difference), then it is fair that in the new ordering the positions of only those two candidates are changed, and ranks of all the others remain unchanged.

(4) *Condition of sovereignty*. For any pair of alternatives there are such two sets of individual preferences, under which the orderings of the alternatives are opposite.

This is the part “If...” of the Arrow’s Theorem. The part “Then...” proclaims the statement receiving name “Theorem of Impossibility”:

*The given requirements are incompatible; i.e. there is no procedure satisfying all four conditions simultaneously.*

That was a sudden surprise (the conditions look so natural and necessary!), and gave rise to hot debates, especially after paraphrasing the Theorem:

*For any election procedure there exists such a set of alternatives, on which the choice can not be made, but the “dictator’s choice” is an exclusion.*

It produced a great noise around the Theorem: “Science proved weakness of democracy!”, “Science shows inevitability of the dictatorship!”, etc. Today the dust has fallen down (many years have passed), and the following comments may be made:

a) Although you may not like it, this is an intrinsic, inherent and unavoidable property of elections, proven mathematically.

b) The Arrow’s Theorem is a statement about voting, not about democracy. They are different things, and its political interpretation is substitution of concepts, confusion of means and purposes.

c) Failure to arrive to a decision results in losses; the loss can be tolerable or inadmissible.

d) If the loss from not making decision is tolerable, we prefer to make decisions collectively, by voting; this gives important dimension to our common activity.

e) If the loss from failure to make decision can not be accepted, we have to exclude the very possibility of not making decisions. And there is the only way to do so – to use a one-sided (“dictator’s”) decision making.

f) The individual choice for a collective problem is neither good nor bad in itself. Everything depends on circumstances. For instance, decline from authoritarian subordinations in the army leads only to lowering its fighting efficiency (as in the Soviet Army with its sharing power between commanders and commissars since 1918 up to 1942). And in an ordinary life we often resort to opinions of a wise man or experts in ambiguous situations.

g) Consideration of properties of voting systems has nothing to do with politics. It is just a rigorous logical study of peculiarities of the algorithms of transformation of a set of individual preferences into one preference agreed to be a collective one. How to exploit the knowledge in real life – this is the politicians’ and managers’ affair.

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